MATH 14-3D2 Spring 2014
Quiz 5 SOLUTIONS

Show all work clearly and in order (on this sheet or an attached sheet) and circle your final answers.
Justify your answers algebraically whenever possible. Work without justification may not receive credit.
You have 25 minutes to take this 10 point quiz.

1. (*3 points*) Find $\frac{dy}{dx}$ when $x^2 + 3xy + 4y^2 = 9x^2y^2$.

Solution: Using implicit differentiation, we compute $\frac{d}{dx}$ of both sides of the given equation to see

$$2x + 3y + 3x\frac{dy}{dx} + 8y\frac{dy}{dx} = 18xy^2 + 18x^2y\frac{dy}{dx}.$$

Therefore if we solve this equation for $\frac{dy}{dx}$ we get

$$\frac{dy}{dx} = \frac{18xy^2 - 2x - 3y}{3x + 8y - 18x^2y}.$$

2. (*3 points*) Suppose that η is a function of α (i.e. $\eta = \eta(\alpha)$). Find $\frac{d\eta}{d\alpha}$ when $\sin(\alpha) + 3\eta\alpha^2 + 5 = \cos(\eta)$.

Solution: Using implicit differentiation, we compute $\frac{d}{d\alpha}$ of both sides of the given equation to see

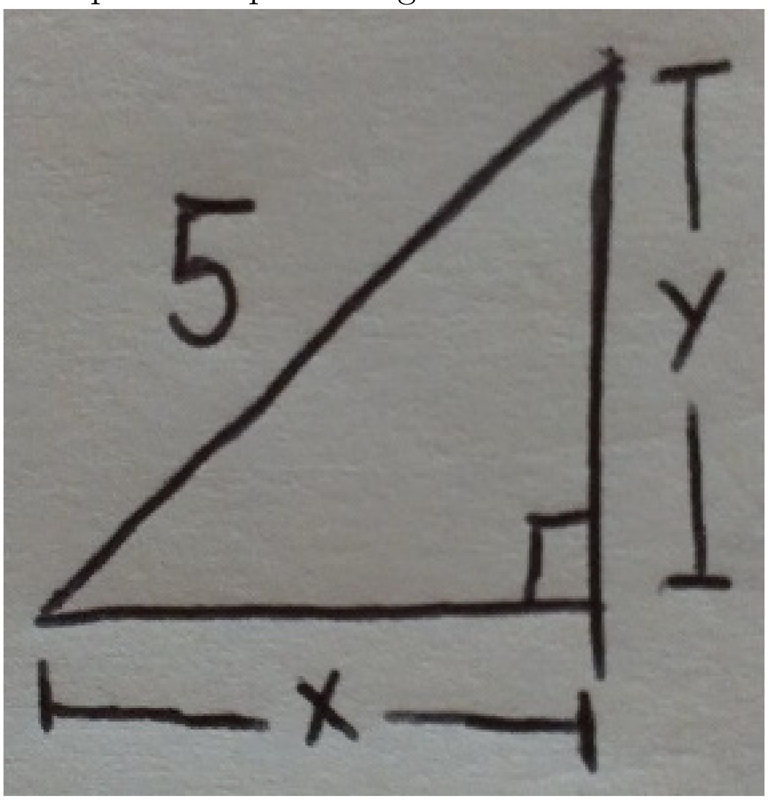
$$\cos(\alpha) + 3\alpha^2\frac{d\eta}{d\alpha} + 6\eta\alpha = -\sin(\eta)\frac{d\eta}{d\alpha}.$$

Therefore if we solve this equation for $\frac{d\eta}{d\alpha}$ we get

$$\frac{d\eta}{d\alpha} = \frac{-\cos(\alpha) - 6\eta\alpha}{3\alpha^2 + \sin(\eta)}.$$

3. (*4 points*) A ladder 5 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of $2\frac{ft}{s}$, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 2 feet from the wall?

a.) Draw a picture representing this situation below.

Solution: 

b.) Label all variables that change with time and the given rate expressed as a derivative with respect to time.

Solution: The variable y denotes the vertical distance from the floor to the top of the ladder. The variable x denotes the horizontal distance from the wall to the bottom of the ladder.

c.) What equation do you need to differentiate for this problem, and what is it after you differentiate?

Before: $x^2 + y^2 = 5$

After: We take $\dfrac{d}{dt}$ of both sides of the above equation and get via implicit differentiation $2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0$

d.) Give me the final answer to the question in sentence form.

Solution: The given rate is $\dfrac{dx}{dt} = -2$. If $x = 2$ then we must conclude via

Pythagorean Theorem that $y = 1$. Hence in the situation described,

$$2(2)(2) + 2(1)\frac{dy}{dt} = 0$$

and so

$$\frac{dy}{dt} = -4.$$

Therefore in the described situation, the ladder is falling down the wall at a rate of $4\frac{ft}{s}$.