Homework 21 Name: Solutions

1. Change each exponential expression to an equivalent expression involving a logarithm.
   a. $9 = 3^2$ $\log_3 9 = 2$
   b. $a^2 = 1.6$ $2 = \log_a 1.6$
   c. $e^x = 8$ $x = \log_e 8$

2. Change each logarithmic expression to an equivalent expression involving an exponential.
   a. $\log_2 8 = 3$ $8 = 2^3$
   b. $\log_a 3 = 6$ $3 = a^6$
   c. $\log_e 4 = x$ $4 = e^x$

3. Solve
   a. $\log_3 x = 2$ $x = 3^2$
   b. $\log_4 64 = x$ already solved!
   c. $e^{3x} = 10$ $3x = \log_e 10 \Rightarrow x = \frac{\log_e 10}{3}$
   d. $\log_6 36 = 5x + 3$ $x = \frac{\log_6 36 - 3}{5}$

4. The pH of a chemical solution is given by the formula: $pH = -\log_{10}[H^+]$ where $[H^+]$ is the concentration of hydrogen ions in moles per liter. Values of pH range from 0 (acidic) to 14 (basic).
   a. What is the pH of a solution for which $[H^+] = 0.1$?
   b. What is the pH of a solution for which $[H^+] = 0.01$?
   c. What seems to happen to pH as hydrogen ion concentration decreases?
   d. Determine the hydrogen ion concentration of an orange with $pH = 3.5$.

5. The formula $D = 5e^{-0.4h}$ can be used to find the number of milligrams D of a certain drug that is in a patient's bloodstream h hours after the drug was administered. When the number of milligrams reaches 2, the drug is to be administered again. What is the time between injections?

6. Write each expression as a single logarithm.
   a. $3\log_5 u + 4\log_5 v \Rightarrow \log_5(u^3) + \log_5(v^4) = \log_5(u^3 \cdot v^4)$
   b. $2\log_3 u - \log_3 v \Rightarrow \log_3(u^2) - \log_3(v) = \log_3\left(\frac{u^2}{v}\right)$
   c. $\log_a(x^2 + 3x + 2) - 2\log_a(x+1) \Rightarrow \log_a(x^2+3x+2) - \log_a((x+1)^2)$
      $= \log_a\left(\frac{x^2+3x+2}{(x+1)^2}\right)$