

4. (18 points) Consider $f(x) = \frac{x}{x-1}$ and $g(x) = x^2 + 3x + 4$. Find the composition functions $f(g(x))$ and $g(g(x))$.

$$f(g(x)) = f(x^2 + 3x + 4) = \frac{x^2 + 3x + 4}{x^2 + 3x + 4 - 1} = \frac{x^2 + 3x + 4}{x^2 + 3x + 3} \quad +3$$

worth 3 pts

$$g(g(x)) = g(x^2 + 3x + 4) = (x^2 + 3x + 4)^2 + 3(x^2 + 3x + 4) + 4$$

fix

forget square (-1)

forget parentheses (-2)

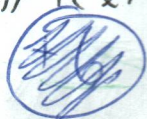
miss a substitution (-2) each

5. (5 points) Find two functions $f(x)$ and $g(x)$ such that $f(g(x)) = \sqrt{\frac{1}{x}}$.

$$f(x) = \sqrt{x}$$

$$g(x) = \frac{1}{x}$$

$$\Rightarrow f(g(x)) = f\left(\frac{1}{x}\right) = \sqrt{\frac{1}{x}}$$



$$\text{or } f(x) = \sqrt{\frac{1}{x}}$$

$$g(x) = x$$

$$f(g(x)) = \sqrt{\frac{1}{x}}$$

getting one of two factors — 2pt

6. (5 points) Find the inverse function of $f(x) = \frac{2x+3}{x+2}$.

$$y = \frac{2x+3}{x+2} \xrightarrow{\text{switch } x+y} x = \frac{2y+3}{y+2}$$

$$\Rightarrow (y+2)x = 2y+3$$

$$\Rightarrow yx + 2x = 2y + 3$$

$$\Rightarrow yx - 2y = 3 - 2x$$

$$\Rightarrow y(x-2) = 3-2x$$

$$\Rightarrow y = \frac{3-2x}{x-2} = f^{-1}(x), \text{ the inverse function of } f(x)$$

+6

switch x and y — 3 pts

solve correctly — 2 pt

sign error — (-1 pt)