

1. Verify that the **values given** are solutions to the given system of equations

a. $\begin{cases} 2x - y = 5 \\ 5x + 2y = 8 \end{cases}; x = 2, y = -1$

$$2(2) - (-1) = 4 + 1 = 5 \quad \checkmark$$

$$5(2) + 2(-1) = 10 - 2 = 8 \quad \checkmark$$

b. $\begin{cases} x - y = 3 \\ -3x + y = 1 \end{cases}; x = -2, y = -5$

$$-2 - (-5) = -2 + 5 = 3 \quad \checkmark$$

$$-3(-2) + (-5) = 6 - 5 = 1 \quad \checkmark$$

c. $\begin{cases} 3x + 3y + 2z = 4 \\ x - 3y + z = 10 \\ 5x - 2y - 3z = 8 \end{cases}; x = 2, y = -2, z = 2$

$$3(2) + 3(-2) + 2(2) = 6 - 6 + 4 = 4 \quad \checkmark$$

$$2 - 3(-2) + 2 = 2 + 6 + 2 = 10 \quad \checkmark$$

$$5(2) - 2(-2) - 3(2) = 10 + 4 - 6 = 8 \quad \checkmark$$

2. Solve each system of equations using substitution. If the system has no solutions, say so.

a. $\begin{cases} x + y = 8 & (1) \\ x - y = 4 & (2) \end{cases}$

$$(2): x = 4 + y$$

$$(1): (4 + y) + y = 8 \Rightarrow 2y + 4 = 8 \Rightarrow 2y = 4$$

$$\Rightarrow \boxed{y = 2} \Rightarrow \boxed{x = 4 + 2 = 6}$$

b. $\begin{cases} 3x - y = 7 & (1) \\ 9x - 3y = 21 & (2) \end{cases}$

c. $\begin{cases} 2x + y = 1 & (1) \\ 4x + 2y = 3 & (2) \end{cases}$

3. Solve each system of equations using matrices. If the system has no solutions, say so.

a. $\begin{cases} 2x + 3y = 6 & (1) \\ 2x - 2y = 1 & (2) \end{cases}$

$$\left[\begin{array}{cc|c} 2 & 3 & 6 \\ 2 & -2 & 1 \end{array} \right] \xrightarrow{R_2 = R_2 - R_1} \left[\begin{array}{cc|c} 2 & 3 & 6 \\ 0 & -5 & -5 \end{array} \right] \rightarrow \begin{cases} 2x + 3y = 6 & (1) \\ -5y = -5 & (2) \end{cases}$$

$$(2): \boxed{y = 1} \quad (1): 2x + 3(1) = 6$$

$$2x = 3 \\ \boxed{x = \frac{3}{2}}$$

b. $\begin{cases} x - y = 6 & (1) \\ 2x - 3z = 16 & (2) \\ 2y + z = 4 & (3) \end{cases}$

c. $\begin{cases} x - 2y + 3z = 7 & (1) \\ 2x + y + z = 4 & (2) \\ -3x + 2y - 2z = -10 & (3) \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 2 & 1 & 1 & 4 \\ -3 & 2 & -2 & -10 \end{array} \right] \xrightarrow{\begin{matrix} R_2 = R_2 - 2R_1 \\ R_3 = R_3 + 3R_1 \end{matrix}} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & 5 & -5 & -10 \\ 0 & -4 & 7 & 11 \end{array} \right]$$

$$\xrightarrow{R_2 = \frac{1}{5}R_2} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & 1 & -1 & -2 \\ 0 & -4 & 7 & 11 \end{array} \right] \xrightarrow{R_3 = R_3 + 4R_2} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 3 & 3 \end{array} \right] \rightarrow \begin{cases} x - 2y + 3z = 7 & (1) \\ y - z = -2 & (2) \\ 3z = 3 & (3) \end{cases}$$

$$\begin{cases} \boxed{z = 1} & (\text{by } 3) \\ \boxed{y = -1} & (\text{by } 2) \\ \boxed{x = 2} & \end{cases}$$